A Procedure for Calculation of Torque Specifications for Bolted Joints with Prevailing Torque

ABSTRACT: This paper presents procedures developed for the calculation of the coefficient of friction of bolt/nut assemblies and for the calculation of torque specifications, which include the case where the fasteners have prevailing torque.

KEYWORDS: fastener torque, prevailing torque, torque calculation, head finish, thread finish

Nomenclature

The following is a list of the nomenclature used. Most terms are consistent with the nomenclature used in the VDI 2230 procedure.

\[ A_0 \] Smallest cross-section area of bolt
\[ A_S \] Effective tensile stress cross-section of the bolt thread per ISO 898-1
\[ D_0 \] Outside diameter of bolt at the smallest cross-section, \( A_0 \) (smaller of \( D_S \) or \( D_T \))
\[ D_2 \] Pitch diameter of bolt thread
\[ D_3 \] Minor diameter of bolt thread
\[ D_{km} \] Effective diameter for friction at the contact of the head of the driven fastener
\[ D_S \] Diameter at stress cross-section \( A_S \)
\[ D_T \] Shank diameter of bolt neck
\[ D_W \] Outside diameter of the contact area under the head of the driven fastener
\[ F_M \] Assembly preload, bolt tensile load at tightening
\[ F_{M,V} \] Assembly preload, bolt tensile load at which the equivalent stress is \( \nu R_{p,0.2} \)
\[ F_{M,MIN} \] Minimum assembly preload expected from tightening to the specified torque
\[ F_{M,MAX} \] Maximum assembly preload expected from tightening to the specified torque
\[ M_A \] Assembly input torque
\[ M_{A,MIN} \] Maximum assembly input torque
\[ M_{A,MAX} \] Minimum assembly input torque
\[ M_{A,PRE} \] Assembly prevailing torque
\[ M_G \] Assembly thread torque, moment in the bolt neck
\[ P \] Pitch of the bolt thread
\[ R_{p,0.2} \] 0.2 \% proof stress of bolt material per ISO 898-1
\[ d_i \] Inside diameter of hollow bolt
\[ d_h \] Inside diameter of the contact area under the head of the driven fastener
\[ \beta_{bh} \] Half flank angle of the bolt thread (\( \pi/6 \) for ISO thread)
\[ \mu_G \] Coefficient of friction between bolt and nut thread
\[ \mu_{G,MIN} \] Minimum coefficient of friction between bolt and nut thread
Introduction

Tightening tests and methods for calculation of the coefficient of friction at the driven fastener’s bearing surface and at the thread contact area are specified in the ISO standard for Fasteners – Torque/Clamp Force Testing (16047) and in the German national standard, Determination of Coefficient of Friction of Bolt/Nut Assemblies Under Specified Conditions (DIN 946). A method for the calculation of torque specifications which requires these friction coefficients is described in the well-known procedure, Systematic Calculation of High Duty Bolted Joints (VDI 2230). These procedures and calculation methods are developed and applicable only for the case where the fasteners have no significant prevailing torque. Prevailing torque is the torque required to turn the driven fastener before any clamping force (or bolt tension) is generated.

In the automobile industry many critical attachments are designed with fasteners that include prevailing torque features, such as all-metal prevailing torque nuts. The error resulting from the application of the standards (referred to above) for calculation of fastener friction and torque specifications has been determined to be of significant magnitude. Therefore, new calculation procedures have been developed and are described in this paper. For the case with no prevailing torque, these same equations can be used but with the value of prevailing torque set to zero.

Theory

Equations for Calculation of Friction

The following equations are developed for calculating the coefficient of head friction ($\mu_K$) and the coefficient of thread friction ($\mu_G$) from data measured during a nut/bolt tightening process. During the tightening process, measured values include input torque ($M_A$), thread torque ($M_G$), and bolt tension ($F_M$).

A mathematical model of the tightening process has been developed by Motosh [1] and is modified here to include the prevailing torque term ($M_{A,PRE}$):

$$F_M = \frac{\left(M_A - M_{A,PRE}\right)}{\left(\frac{P}{2\pi} + \frac{\mu_G D_2}{2 \cos \beta_{ih}} + \frac{\mu_K D_{km}}{2}\right)}$$

(1)

Where:
This equation is rearranged to solve for input torque, which is reacted by the torque under the head of the driven fastener (head torque) and the torque at the contact of the nut and bolt threads (thread torque):

\[
M_A = M_{A,PRE} + F_M \left( \frac{P}{2\pi} + \frac{\mu_G D_2}{2 \cos \beta_{th}} \right) + F_M \left( \frac{\mu_K D_{km}}{2} \right)
\]

And the head torque is the difference of input torque minus thread torque:

\[
M_A - M_G = F_M \left( \frac{\mu_K D_{km}}{2} \right)
\]

Equation 4 is rearranged to solve for thread friction:

\[
\mu_G = \frac{2 \cos \beta_{th}}{D_2} \left( \frac{M_G - M_{A,PRE}}{F_M} - \frac{P}{2\pi} \right)
\]

Equation 5 is rearranged to solve for head friction:

\[
\mu_K = \frac{2(M_A - M_G)}{D_{km} F_M}
\]

And finally, if head friction and thread friction are assumed to be equal, then \(\mu_{ges}\) is substituted for \(\mu_G\) and \(\mu_K\) in Eq 3, and the equation is rearranged to solve for friction:

\[
\mu_{ges} = \left( \frac{M_A - M_{A,PRE}}{F_M} - \frac{P}{2\pi} \right) \left( \frac{D_2 - D_{km}}{2 \cos \beta_{th}} \right)
\]

Equations 6 and 7 are used to calculate the coefficients of head and thread friction from the measurement of prevailing torque, input torque, and thread torque, at a selected value of bolt tension during assembly of fasteners in a laboratory test.
Equations for Calculation of Torque Specification

The following equations are developed for calculating a torque specification, utilizing the coefficients of head and thread friction calculated using equations above. During the assembly of the fasteners, the bolt shank and threaded section are stressed in tension and additionally in shear due to the applied torque. The equations are developed to allow for the calculation of an upper torque specification limit that will result in a desired maximum equivalent stress in the bolt shank due to the tension and shear combined stresses. The minimum torque specification is calculated to result in a specified tolerance so that the tightening process will be statistically capable for a selected tightening tool. For example a torque specification tolerance of ±15 % might be required to have a capable process with a selected mechanical clutch shutoff tool.

The equivalent stress in the bolt due to the tensile stress and the torsional stress (from the maximum distortion energy theory of failure) is:

$$\sigma_e = \sqrt{\sigma_t^2 + 3\tau^2}$$  \hspace{1cm} (9)

The desired magnitude for the maximum equivalent stress resulting from tightening is:

$$\sigma_e = \nu R_{p,0.2}$$ \hspace{1cm} (10)

The tensile stress is:

$$\sigma_t = \frac{F_{M,v}}{A_0}$$ \hspace{1cm} (11)

And the torsional stress is:

$$\tau = \frac{16M_GD_0}{\pi(D_0^4 - d_i^4)}$$ \hspace{1cm} (12)

Substituting Eqs 10, 11, 12 and 4 into Eq 9 and solving for $F_{M,v}$ yields the following equation. Here, the value $F_{M,v}$ is the allowable magnitude for the bolt preload such that the equivalent stress is $\nu R_{p,0.2}$ for any value of thread friction, $\mu_G$.

$$F_{M,v} = -6M_{A,PRE}K_1K_2 + \sqrt{\left(6M_{A,PRE}K_1K_2^2 - 4\left(1 + 3K_1K_2^2\right)\frac{3M_{A,PRE}^2K_1 - K_3}{2\left(1 + 3K_1K_2^2\right)}\right)^2}$$ \hspace{1cm} (13)

Where:

$$K_1 = \left[\frac{4D_0}{(D_0^2 + d_i^2)}\right]^2, \hspace{0.5cm} K_2 = \left[\frac{P}{2\pi} + \frac{\mu_GD_2}{2\cos\beta_{th}}\right], \hspace{0.5cm} \text{and} \hspace{0.5cm} K_3 = \left[\frac{\nu R_{p,0.2}\nu(D_0^2 - d_i^2)}{4}\right]^2$$

For the case where the minimum bolt cross-section is the threaded section:

$$D_0 = \frac{D_2 + D_3}{2}$$ \hspace{1cm} (14)
And for the case where the minimum bolt cross section is the shank:

\[ D_0 = D_T \]  \hspace{1cm} (15)

If the upper and lower limits of the torque specification are \( M_{A,\text{MAX}} \) and \( M_{A,\text{MIN}} \), respectively, then the torque specification tolerance is:

\[ M_{A,\text{TOL}} = \frac{(M_{A,\text{MAX}} - M_{A,\text{MIN}})}{2} \]  \hspace{1cm} (16)

The nominal of the torque specification is:

\[ M_{A,\text{NOM}} = \frac{(M_{A,\text{MAX}} + M_{A,\text{MIN}})}{2} \]  \hspace{1cm} (17)

The tolerance in terms of a percentage of nominal is:

\[ M_{A,\text{TOL\%}} = \frac{M_{A,\text{TOL}}}{M_{A,\text{NOM}}} \]  \hspace{1cm} (18)

In order to calculate the maximum bolt preload (\( F_{M,\text{MAX}} \)) so that the equivalent stress does not exceed the value \( \nu R_{p,0.2} \), the minimum value of thread friction is substituted into Eq 13:

\[ F_{M,\text{MAX}} = -6M_{A,\text{PRE}}K_1K_2 + \sqrt{(6M_{A,\text{PRE}}K_1K_2)^2 - 4\left(1 + 3K_1K_2^2\right)^3M_{A,\text{PRE}}^2K_1 - K_3} \]  \hspace{1cm} (19)

Where:

\[ K_1 = \left[ \frac{4D_0}{D_0^2 + d_i^2} \right]^2, \quad K_2 = \left[ \frac{P}{2\pi} + \frac{\mu_{G,\text{MIN}}D_2}{2\cos\beta_{th}} \right], \quad \text{and} \quad K_3 = \left[ \frac{\nu R_{p,0.2}\pi(D_0^2 - d_i^2)}{4} \right]^2 \]

The maximum torque is calculated by substituting Eq 16 into Eq 3 with minimum values of head and thread friction:

\[ M_{A,\text{MAX}} = M_{A,\text{PRE}} + F_{M,\text{MAX}} \left[ \frac{P}{2\pi} + \frac{\mu_{G,\text{MIN}}D_2}{2\cos\beta_{th}} + \frac{\mu_{K,\text{MIN}}D_{km}}{2} \right] \]  \hspace{1cm} (20)

The minimum torque is calculated from the maximum based on the desired torque tolerance:

\[ M_{A,\text{MIN}} = M_{A,\text{MAX}} \left( \frac{1 - M_{A,\text{TOL\%}}}{1 + M_{A,\text{TOL\%}}} \right) \]  \hspace{1cm} (21)
And finally, the bolt minimum preload when assembled to the minimum torque is calculated by substituting \( M_{A,MIN} \) into Eq 1 with maximum values of head and thread friction:

\[
F_{M,MIN} = \frac{(M_{A,MIN} - M_{A,PRE})}{\frac{P}{2\pi} + \frac{\mu_{G,MAX} D_2}{2 \cos \beta_{th}} + \frac{\mu_{K,MAX} D_{km}}{2}}
\]  

(22)

**Discussion**

**Error Due to Misapplication of Standard Equations**

The equations used for the calculation of friction in ISO 16047 [1] and in the German national standard DIN 946 [2] are:

\[
\mu_G = \frac{2 \cos \beta_{th}}{D_2} \left( \frac{M_G}{F_M} - \frac{P}{2\pi} \right)
\]  

(23)

and:

\[
\mu_K = \frac{2(M_A - M_G)}{D_{km} F_M}
\]  

(24)

The root of these equations is the long form equation that describes the relationship between torque and tension during the assembly of a fastener:

\[
F_M = \frac{M_A}{\left( \frac{P}{2\pi} + \frac{\mu_G D_2}{2 \cos \beta_{th}} + \frac{\mu_K D_{km}}{2} \right)}
\]  

(25)

For a given bolt/nut assembly:

\[
\frac{1}{\left( \frac{P}{2\pi} + \frac{\mu_G D_2}{2 \cos \beta_{th}} + \frac{\mu_K D_{km}}{2} \right)} = K \text{ (Constant)}
\]

and Eq 25 reduces to:

\[
F_M = KM_A
\]  

(26)

This is an equation for a line that passes through the origin of the input torque versus bolt tension graph. The point here is explained in Fig. 1. The line labeled “actual” represents the torque/tension relationship for a bolt/nut assembly with prevailing torque, \( M_{A,PRE} \). If the friction is calculated based on values of \( M_A \) and \( M_G \) at the bolt tension \( F_{M,1} \) using Eqs 23 and 24, then the analytical description of the torque/tension relationship, based on Eq 25, is shown by the line labeled “analytical.” As shown in the figure, the calculated value of bolt tension at \( M_A \) is \( F_{M,3} \), and the actual value of bolt tension is \( F_{M,2} \), resulting in the error shown. Also, note that the error is zero exactly at the value of tension at which the head and thread friction were calculated.

For some top-lock all-metal prevailing torque nuts, the error between the actual and calculated values of \( F_{M,MIN} \) is approximately 15%.
Discussion of Method for Torque Specification Calculation

Figure 2 shows the process described above for the calculation of torque specifications. The curved line labeled $F_{M,\text{MAX}}$ shows the values of bolt preload that result in the desired equivalent stress $\nu R_{P0.2}$ for various values of $\mu_G$. The two bold lines that intersect the torque-axis at $M_{A,\text{PRE}}$ describe the limits of the bolt/nut tightening process, where the under head friction varies from $\mu_{K,\text{MIN}}$ to $\mu_{K,\text{MAX}}$, and the thread friction varies from $\mu_{G,\text{MIN}}$ to $\mu_{G,\text{MAX}}$. The maximum of the torque specification $M_{A,\text{MAX}}$ is at the intersection of the $F_M$ line, and the line that describes the bolt/nut tightening process is at the minimum values of friction. So, at the maximum of the torque specification, and with fasteners that have minimum friction at both the head and thread, the bolt equivalent stress is at the desired maximum value. $M_{A,\text{MIN}}$ is calculated from $M_{A,\text{MAX}}$ to achieve a desired torque tolerance for process capability. And finally, $F_{M,\text{MIN}}$ is at the intersection of $M_{A,\text{MIN}}$, and the line that describes the bolt/nut tightening process is at the maximum values of friction.

Conclusion

Equations 20 and 21 can be used for the calculation of torque specifications for the case of fasteners with prevailing torque. The resulting maximum and minimum bolt preload is calculated with Eqs 19 and 22. When these equations are used, the fastener head and thread friction must be calculated per Eqs 6 and 7. The use of these equations can result in a 15 % improvement in the accuracy of torque specification calculation, when compared to the results of calculations that do not consider the prevailing torque.
FIG. 2—Diagram of procedure for calculation of torque specification.

References